University of Diyala
College of Engineering
Department of Materials

Fundamentals of Electric Circuits

Lecture Eight

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2-13 Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 1. How do we combine resistors $R_1$and R_6 through when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 1

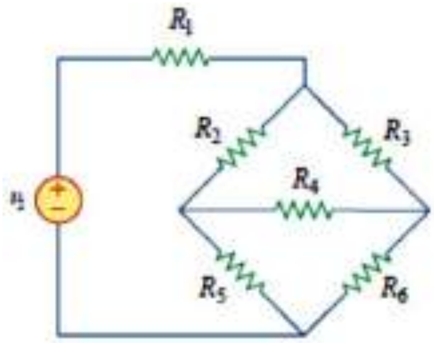

Fig. 1

Can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2 and the delta (Δ) or pi (Π) network shown in Fig. 3. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

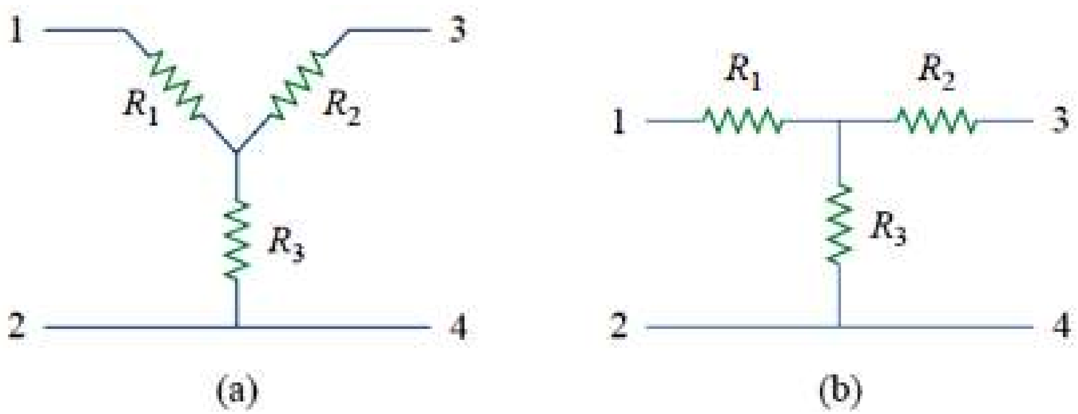

Fig. 2

Two Forms of the same network: (a) Y, (b) T.

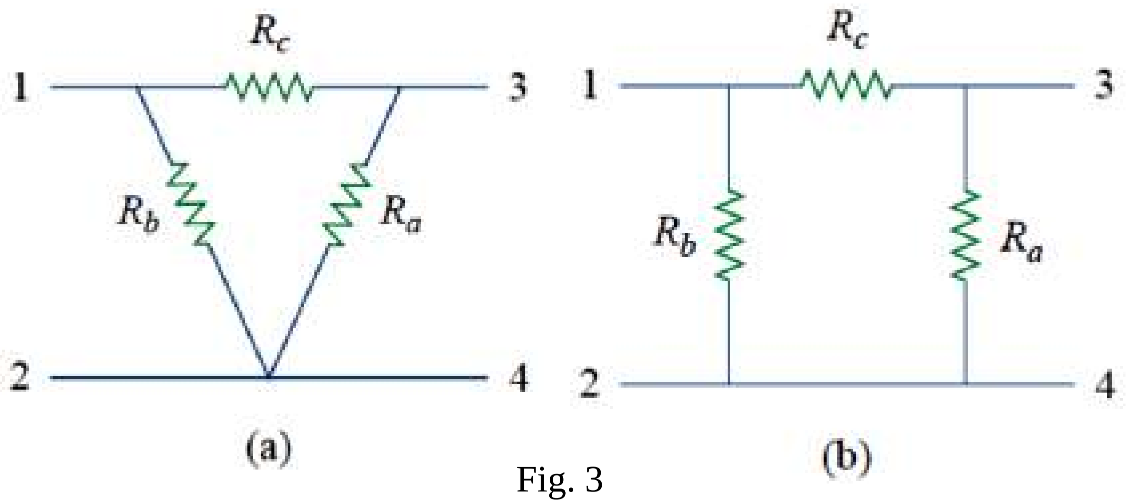

Fig. 3

Two Forms of the same network: (a) Δ, (b) Π.

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2 and 3, for example,

$$R_{12}(Y) = R_1 + R_3 \qquad (1)$$

$$R_{12}(\Delta) = R_b \| (R_a + R_c)$$

Setting $R_{12}(Y) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \qquad (2a)$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \qquad (2b)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \qquad (2c)$$

Subtracting Eq. (2c) from Eq. (2a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \qquad (3)$$

Adding Eqs. (2b) and (3) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad (4)}$$

And subtracting Eq. (3) from Eq. (2b) yields

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad (5)}$$

Subtracting Eq. (4) from Eq. (2a), we obtain

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad (6)}$$

We do not need to memorize Eqs. (4) to (6). To transform a network to Y, we create an extra node n as shown in Fig. 2.49 and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

One can follow this rule and obtain Eqs. (4) to (6) from Fig. 4.

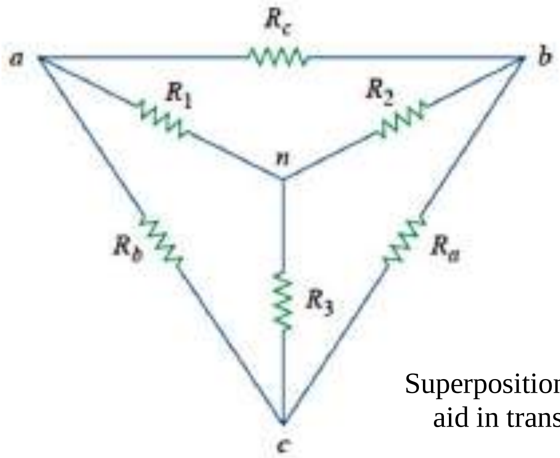

Fig. 4
Superposition of Y and networks as an aid in transforming one to the other.

Wye to Delta Conversion

To obtain the relationships necessary to convert from a Y to a Δ, first divide Eq. (6) by Eq. (4):

$$\frac{R_3}{R_1} = \frac{(R_a R_b)/(R_a + R_b + R_c)}{(R_b R_c)/(R_a + R_b + R_c)} = \frac{R_a}{R_C}$$

Or

$$R_a = \frac{R_c R_3}{R_1}$$

Then divide Eq. (6) by Eq. (5)

$$\frac{R_3}{R_2} = \frac{(R_a R_b)/(R_a + R_b + R_c)}{(R_a R_c)/(R_a + R_b + R_c)} = \frac{R_b}{R_c}$$

Or

$$R_b = \frac{R_c R_3}{R_2}$$

Substituting for R_a and R_b in Eq. (5) yields

$$R_2 = \frac{(R_c R_3/R_1)R_c}{(R_3 R_c/R_2) + (R_c R_3/R_1) + R_c}$$

$$= \frac{(R_3/R_1)R_c}{(R_3/R_2) + (R_3/R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_c/R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)}$$

$$= \frac{R_2 R_3 R_c}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

And

$$\boxed{R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} \qquad (7)}$$

We follow the same procedure for R_b and R_a:

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} \qquad (8)$$

And

$$R_b = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \qquad (9)$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be balanced when

$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta,$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \; or \; R_\Delta = 3R_Y$$

Example 1: Convert the Δ network in Fig. 5a to an equivalent Y network

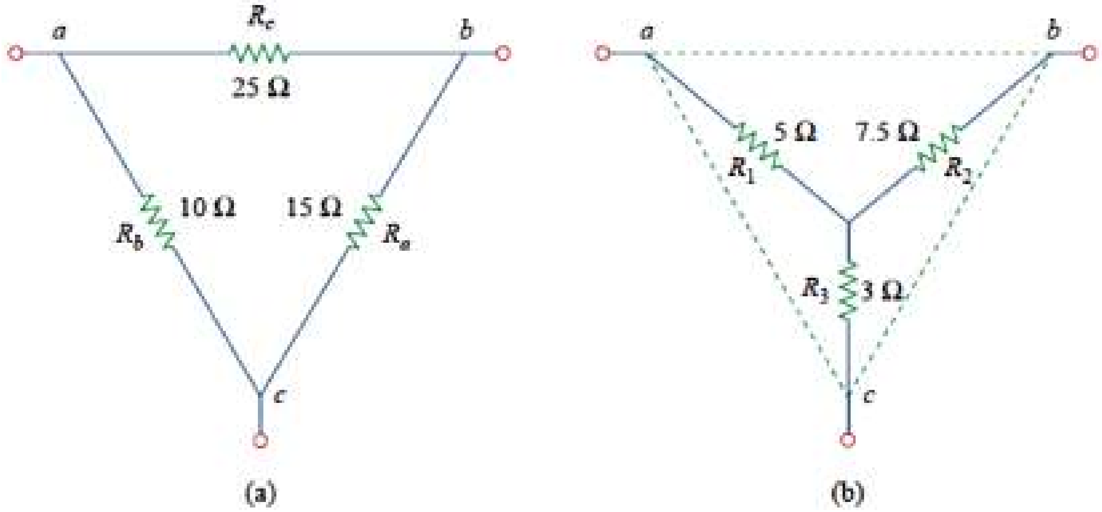

Fig.5

For Example 5: (a) original Δ network, (b) Y equivalent network.

Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{15 + 10 + 25} = \frac{375}{50} = 7.5\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{10 \times 15}{15 + 10 + 25} = \frac{150}{50} = 3\Omega$$

The equivalent Y network is shown in Fig. 5(b).

Example 2: Obtain the equivalent resistance R_{ab} for the circuit in Fig. 6 and use it to find current i.

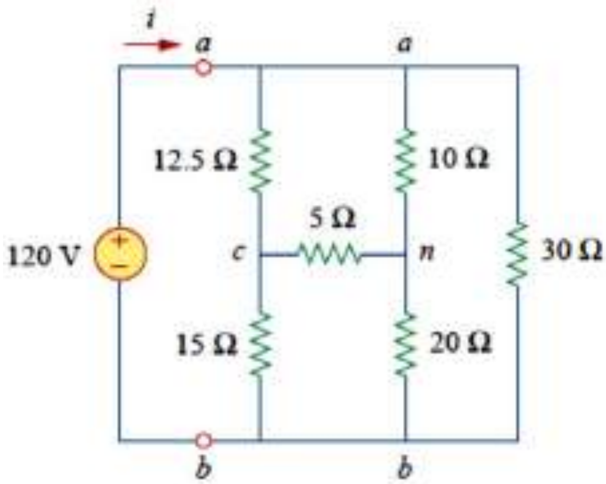

Fig. 6

H.W. 1:For the bridge network in Fig. 7, find R_{ab} and i.

$Answer: 40\Omega, 2.5\ A.$